

A Multi-objective Approach for Routing and Wavelength Converter Allocation Under Uncertainty

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Abstract—Wavelength Division Multiplexing (WDM) networks are designed for long-term operations in which the uncertainty of future traffic plays an important role. The performance of these networks is highly dependent on the routing and the wavelength-converter allocation algorithms. In order to achieve a good performance on a long-term basis, both problems have to be dealt with together and traffic uncertainty should be included. To this aim, this work proposes a joint optimization approach where the converters allocation plan and paths for routing are calculated simultaneously. A Multi-Objective Evolutionary Algorithm (MOEA) is proposed to minimize the number of wavelength converters, the average blocking probability, and an unfairness measure, all three at the same time, the uncertainty is modelled by means of a set of scenarios. The proposed MOEA calculates an approximation to the optimal set of solutions. This work presents experimental results showing the feasibility of the proposed approach in a multi-objective and uncertain context.

I. INTRODUCTION

Wavelength Division Multiplexing (WDM) networks using wavelength-routing have emerged as a feasible alternative for Wide Area Network (WAN) [22]. A WDM network consists of wavelength cross-connect nodes interconnected by optical fibres. This network provides all-optical communication between pairs of nodes by establishing circuit-switched connections called *light-path* [24].

The Routing problem when jointly treated with the Wavelength Converter Allocation (WCA) problem is known as *Routing and Wavelength Converter Allocation (RWCA)* problem. This problem is critical to get a good performance in WDM networks. The Routing and WCA problems have been traditionally treated in the literature one after the other in what is known as *iterative-joint* method [7], [9], [20], [25].

Recently, we have shown that a *full-joint* optimization approach may be a better way to solve the problem [17], [18]. It should be also mentioned that, to the best of our knowledge, no previous work solving the wavelength converter allocation problem has yet considered uncertain traffic, i.e. all analyzed works have only considered one dynamic pattern of traffic at a time; however, the network performance, even if very good at the beginning, can get worse when traffic pattern changes. Of course, in this new context, it is necessary to deal with traffic uncertainty. Therefore, for the first time in the literature, this work studies the RWCA problem using a *full-joint* approach under uncertain traffic. In addition, given that the cost and

blocking probability are in conflict, we treat this problem in a pure multi-objective context, considering also the unfairness of feasible solutions.

To solve the above mentioned multi-objective RWCA problem under traffic uncertainty, a *Multi-Objective Evolutionary Algorithm* (MOEA) [6] is proposed. In this context, a MOEA algorithm calculates the best wavelength converter allocation combined with the best routes minimizing the number of converters as well as the average blocking probability and an unfairness measure on the blocking probability in order to get a set of good trade-off solutions [17].

II. UNCERTAIN TRAFFIC IN NETWORKS

Network design under uncertain traffic has been studied considering mainly two basic characteristics: (a) model of uncertain traffic [16], [2], and (b) traffic dynamics [1]. In this context, state-of-the-art works have reported three traffic models to study communications networks [2]: model based on scenarios, hose, and polyhedral models. This work addresses the scenarios-based model that can be implemented by given a set of traffic matrices to represent the traffic's uncertainty.

Sean et al. [21] dealt with the problem of capacity expansion of links considering the minimization of the number of blocked requests, subject to a total capacity expansion constraint. They proposed a two-level stochastic lineal programming approach. Due to the high complexity of the problem, they used a sampling-based approach.

Lisser et al. [14] and also Riis and Andersen [19] studied the link capacity sizing problem for telecommunication networks. They treated this problem as a two-level stochastic programming problem minimizing investment costs and the number of blocked requests using an iterative-joint approach.

Kennington et al. [12] applied a robust optimization model [15] to the link capacity sizing problem in WDM optical networks. The benefit of this model was compared to other techniques such as: average value, two-level stochastic programming and the worst case. Note that in these last works [14], [19], [12], the fractional routing model was considered; therefore, the complexity was very high and simulations were limited to a small number of requests.

Grosso et al. [8] dealt with the virtual topology design for wavelength-routed networks. An approach based on Tabu

Search was used in order to balance the traffic load of different wavelengths where uncertain traffic was modelled by scenarios.

In short, a number of interesting approaches to deal with traffic uncertainty have been proposed, however, no previous work has treated the RWCA problem under uncertain traffic. Therefore, this work studies for the first time the RWCA problem under uncertain traffic which is modeled by scenarios, a technique to study uncertainty highly used in the literature [21], [14], [19], [12], [8].

III. ROUTING AND WAVELENGTH CONVERTER ALLOCATION

The complexity of the converter allocation problem has already been largely studied [3], [13], [26], [27]. In particular, it was proven that the problem belongs to the NP-Complete class [27], being at least as difficult as the minimal vertex cover problem [13].

Harai et al. [10] studied the wavelength converter allocation problem for the first time. They proposed a heuristic that minimizes the average blocking probability in two steps: first, a given number of converters are located and next, a route for each request is selected from an alternative paths' table. The paths' table consists of K -alternative paths for each pair of source-destination nodes. The table is pre-calculated before the optimization takes place. In [23], the converter allocation problem is treated considering a uniform and link independent load. Besides, a Dynamic Programming algorithm for non-uniform link load was developed.

Chu et al. [4] studied the performance of WCA, Routing and Wavelength Assignment (RWA) algorithms. They showed experimentally that a converter allocation algorithm should be designed as a function of the adopted RWA algorithm. Two RWA algorithms were developed for fixed-alternative routing and low-load routing. It is important to note that, Karasan and Ayanoglu [11] got analogous conclusions on the interaction between routing algorithms and wavelength assignment algorithms. Considering these results, the following conjecture may be derived: the wavelength converter allocation, the routing, and the wavelength assignment problems should be designed together in order to obtain a high performance network.

A new approach considering a multi-objective optimization context was proposed in [17], [18]. In particular [17] proposes a Multi-objective Evolutionary Algorithm (MOEA) to solve the converter allocation problem for sparse-partial-wavelength-conversion network where the blocking probability is calculated by expensive simulations. A more recent work [18], also proposes a MOEA approach in order to solve the routing and the converter allocation problems simultaneously for sparse-wavelength-conversion network. In this work, the blocking probability is analytically calculated.

Analysing previous reports, we can observe that the RWCA problem has been dealt with by approaches that can be classified according to the type of traffic, i.e., static or dynamic [28]. Also, a classification based on the method used to calculate the blocking probability is possible [20]: simulation and analytical

approaches. According to [17], algorithms can be designed for mono- or multi-objective optimization contexts. In general, we also can classify the previous works as: (1) iterative or (2) full-joint optimization approaches [18]. Note that, no work of the state-of-the-art has considered traffic uncertainty in the RWCA problem. Therefore, this work proposes to model the RWCA problem following the approach of Pinto-Roa et al. [18], including, this time, uncertain traffic.

IV. MULTI-OBJECTIVE FORMULATION FOR RWCA

For the sake of completeness, the nomenclature and basic symbols used in this work, are next defined:

$ \cdot $	Cardinality of a set;
\mathbf{G}	Direct graph representing a network topology;
\mathbf{V}	Set of nodes in \mathbf{G} ;
\mathbf{L}	Set of links in \mathbf{G} ;
N	Number of network nodes, i.e., $N = \mathbf{V} $;
s	Source node index of a request, $s \in \mathbf{V}$;
d	Destination node index of a request, $d \in \mathbf{V}$;
\mathbf{a}	A solution vector for the converter allocation, $\mathbf{a} = [a_1, a_2, \dots, a_N]$, where $a_i \in \{0, 1\}$. If $a_i = 1$ then node i is equipped with a converter, otherwise no converter is assigned to node i ;
$r_{s d}$	Request from source node s to destination node d ;
$t_{s d}$	Traffic load of a request $r_{s d}$ measured in Erlang;
\mathbf{T}	Traffic matrix for all-to-all requests; $\mathbf{T} = \{t_{s d}\}$; $\forall s, d$ and $s \neq d$;
Ω	Number of scenarios;
Γ	Set of traffic matrices $\Gamma = \{\mathbf{T}_w\}$ with $w = 1, 2, \dots, \Omega$; Γ represents the set of scenarios;
K	Number of alternative paths per request, even through only one path will be assigned to the request;
$\mathcal{P}_{k s d}$	k th path associated to request $r_{s d}$; $\mathcal{P}_{k s d} = \{i_0, i_1, \dots, i_u\}$ where $i_v \in \mathbf{V}$ ($0 \leq v \leq u$), $i_0 = s$, $i_u = d$, and u is the number of links of $\mathcal{P}_{k s d}$;
\mathbf{P}	Paths' table for all-to-all requests with K -alternative paths per request; $\mathbf{P} = \{\mathcal{P}_{k s d}\}$;
$P_{s d}$	End-to-end blocking probability for request $r_{s d}$ calculated according to [18];
\mathbf{K}	A solution matrix of dimension $N \times N$ for the routing problem, $\mathbf{K} = \{k_{s d}\}$, where $k_{s d} \in \{1, 2, \dots, K\}$. $k_{s d}$ indicates the route number $k_{s d}$ of \mathbf{P} used to transmit information from source node s to destination d . Obviously, if $s = d$, $\mathcal{P}_{k s d}$ is an empty route. Note that $k_{s d}$ represents the index of the actual selected path among the K available paths;

This work proposes to solve the RWCA problem as a multi-objective optimization problem [5] using an analytical approach to compute the success probability. Given a topology \mathbf{G} , a set of traffic matrices Γ and a paths' table \mathbf{P} for requests, the goal is to calculate a set of solutions that minimizes the *Number of Converters* $f_1(\mathbf{x})$, *Average Blocking Probability (ABP)* $f_2(\mathbf{x})$, and *Unfairness* $f_3(\mathbf{x})$:

$$\text{Minimize } \mathbf{f}(\mathbf{x}) = [f_1(\mathbf{x}), f_2(\mathbf{x}), f_3(\mathbf{x})] \quad (1)$$

where

$$f_1(\mathbf{x}) = \sum_{\forall i \in \mathbf{V}} a_i, \quad (2)$$

$$f_2(\mathbf{x}) = \frac{1}{\Omega} \cdot \sum_{\omega=1}^{\Omega} P_B^\omega(\mathbf{a}, \mathbf{K}) \quad (3)$$

and

$$f_3(\mathbf{x}) = E(\mathbf{a}, \mathbf{K}) = \max_{\omega=1}^{\Omega} \{\Delta E_\omega(\mathbf{a}, \mathbf{K})\}, \quad (4)$$

subject to:

$$a_i \in \{0, 1\}, \quad \forall i \in \mathbf{V} \quad (5)$$

$$k_{sd} \in \{1, 2, \dots, K\}, \quad \forall s, d \in \mathbf{V} \text{ and } s \neq d \quad (6)$$

where:

- expression (5) assures that a network node has assigned a converter or none. At the same time, (6) restricts the number of alternative paths for each source-destination nodes;
- $f_2(\mathbf{x})$ and $f_3(\mathbf{x})$ are calculated using the system model introduced in [18], where a solution $\mathbf{x} = (\mathbf{a}, \mathbf{K})$ defines what nodes use converters (using vector \mathbf{a}) and what paths (represented by matrix \mathbf{K}) are selected from \mathbf{P} for routing;
- $P_B^\omega(\mathbf{a}, \mathbf{K}) = [\prod_{\forall r,s,d} (P_{sd}^w)^{t_{sd}}]^{(\sum_{\forall r,s,d} t_{sd})^{-1}}$ is the weighed geometric average considering all the end-to-end success probability at scenario w [18];
- $\Delta E_\omega(\mathbf{a}, \mathbf{K}) = P_{max}^\omega - P_{min}^\omega$ is the unfairness at scenario w ; where $P_{max}^\omega = \max_{\forall r,s,d} \{P_{sd}^\omega\}$ and $P_{min}^\omega = \min_{\forall r,s,d} \{P_{sd}^\omega\}$.

We can see how uncertainty is taken into account through the objectives (3) and (4) in such a way that a more robust solution will be the one that minimizes, at the same time, both objective functions. To the best of the authors knowledge this is the first time the RWCA problem is modelled taking into account traffic uncertainty.

V. EXPERIMENTAL SETUP AND RESULTS

This section presents experimental results to show that it is possible to solve the RWCA problem under uncertain traffic in a pure multi-objective context, using a MOEA.

To simplify the presentation of simulations that follow, we only consider three paths ($K = 3$), following recommendations given in [18] for one scenario. Presented results correspond only to the NSF topology shown in Figure 1, even though larger topologies were also simulated by the authors.

Additionally, this work studies the quality of non-dominated solutions with respect to the number of converters given that this number also affects the average blocking probability and the unfairness of a solution as shown in what follows.

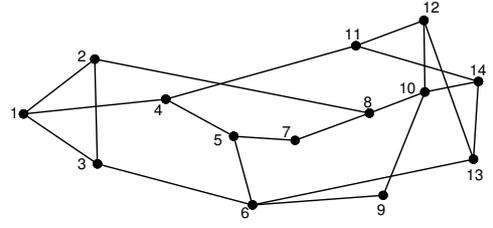


Figure 1: NSF Topology with 14 nodes and 21 links.

A. Proposed Algorithm and Parameters

The NSGA-II evolutionary algorithm was adapted and implemented following the ideas in [18]. The main differences are in the objective functions where, in this work, uncertainty is taken care by means of equations (3) and (4). The input to the algorithm is given by the network topology, traffic matrices, and the number of available wavelengths. The algorithm provides as its output a set of nondominated solutions indicating the location of each converter and the routing to follow in each request.

The evolutionary algorithm parameters used in the reported simulations were configured with: (a) 100 individuals as population size, (b) a crossover probability of 100%, (c) a mutation probability of 30%, and (d) 20000 evaluations of the objective functions as stop criterion. The initial population was generated randomly. The chromosome and evolutionary operators were adopted from [18]. The chromosome structure is conformed by two genes. The first one is a vector of N binary values $\{0, 1\}$ representing converters allocation (\mathbf{a}) while the second gene is another vector of size $N \times (N-1)$ that represents the routing path between each pair of nodes (\mathbf{K}), as explained in detail in [18]. Thus, the final chromosome size is $N + N \times (N-1)$.

B. Experimental Environment

Simulations were performed on a computer with 12 Intel Xeon L7455 processors, working at 2.13 GHz with 32 GB of RAM. The implemented algorithm for the RWCA problem was the NSGA-II [6], according to the proposal made in [18]. The NSGA-II algorithm was implemented in Java 1.6, under SENT OS Release 5.4. As stated, results are presented only for the NSF network topology with 14 nodes and 21 links, as shown in Figure 1.

The following conditions were considered to define the architecture of the WDM network used in the simulations: (a) WDM network based on mono-fibre architecture [22], (b) eight wavelengths ($W=8$) per optical fibre [11], (c) traffic matrices \mathbf{T} are all-to-all, and (d) wavelengths are randomly assigned to each converter node when needed [18].

The outline of the experiment is summarized by the following steps:

- 1) *Traffic scenarios.* Tests with 25, 50, 75 and 100 traffic matrices were generated randomly for low load (0% ~ 30%), medium load (30% ~ 60%) and high load (60% ~ 100%) scenarios.

$\sim 90\%$). The maximum traffic in any fibre is W Erlang; i.e. there are W wavelength channels per fibre.

2) *NSGA-II runs*. For each instance of the problem, the following steps were performed:

- a) The NSGA-II algorithm was run 10 times to obtain 10 solutions sets $\mathbf{PS}(i)$ which were approximations of the Pareto set $\mathbf{PS}_{true}; i = \{1, 2, \dots, 10\}$.
- b) These 10 sets of solutions were joined to obtain a unique set: $\mathbf{SF} = \bigcup_{i=1}^{10} \mathbf{PS}(i)$.
- c) Dominated solutions were eliminated to calculate a non-dominated set of solutions or Pareto set approximation called $\mathbf{PS} \subseteq \mathbf{SF}$.

Finally, the solution set \mathbf{PS} is returned as the final set to be considered as the best approximation to the Pareto set calculated by the NSGA-II algorithm.

C. Numerical Results

Considering the notation and the experimental criteria above described, the non-dominated fronts obtained for different traffic loads and various number of scenarios are presented in figures 2, 3, and 4. Due to space limitations these figures do not include the results for 25 and 75 scenarios. The computation times (in hours), to calculate those solutions (\mathbf{PS}), are shown in Table I.

Load	Scenarios			
	25	50	75	100
low	0.60	1.2	1.7	2.4
median	0.57	1.1	1.7	2.3
high	0.50	1.0	1.5	2.0

Table I: Total calculation time (in hours) for 10 runs and all different traffic loads and number of scenarios.

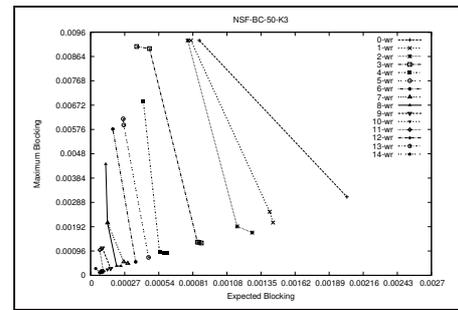
Note that all solutions with the same number of converters form a cluster, as shown in figures 2, 3, and 4. In each cluster, the average of blocking probability and unfairness are clearly in trade-off, i.e. it is not possible to improve one without deteriorating the other. In general, when the number of converter increases, the number of found solutions decreases.

Another interesting aspect to point out is that the non-dominated solutions are close to each other and near to the (0,0) coordinates, for low load. However, this is not the case when traffic load increases. Also note that the main structural characteristics of non-dominated solutions remain mostly the same for any load level.

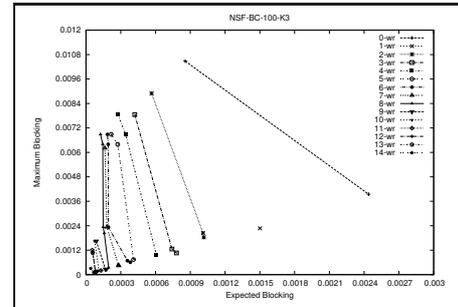
Finally, a relevant aspect of the presented experimental results is that the quality of nondominated solutions improves when the number of converter increases; i.e. at least one solution of any n -converter set dominates a solution of an m -converter set, whenever $m < n$.

VI. CONCLUSIONS AND FUTURE WORK

This work is the first to introduce the Multi-objective Routing and Wavelength Converter Allocation Problem for WDM optical networks under traffic uncertainty. Three different objectives are considered: (1) number of wavelength converters, (2) average blocking probability and, (3) unfairness.

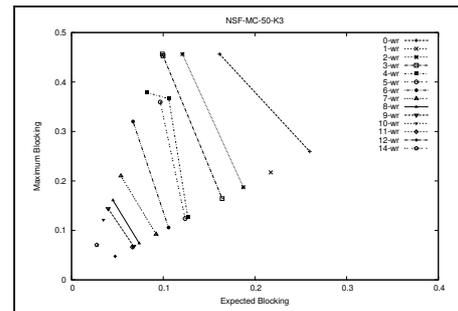


(a) 50 scenarios

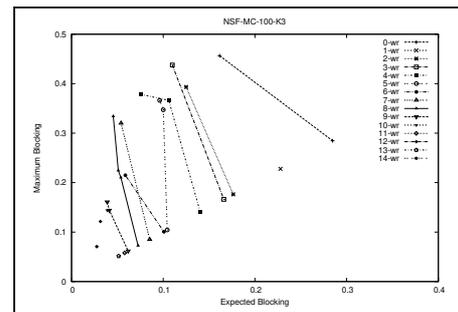


(b) 100 scenarios

Figure 2: Non-dominated solutions with low load.



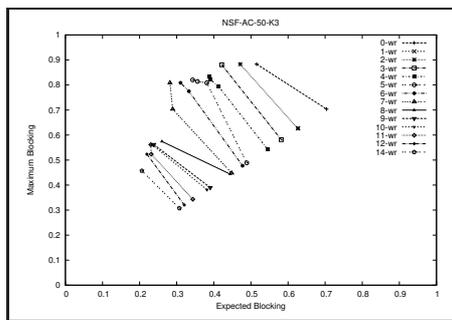
(a) 50 scenarios



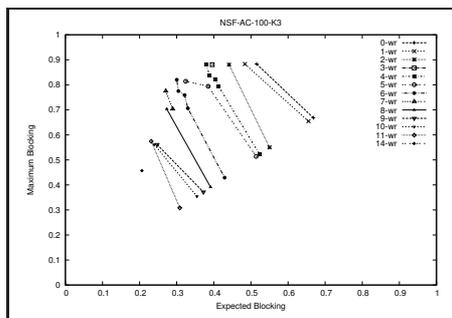
(b) 100 scenarios

Figure 3: Non-dominated solutions with medium load.

Given the novelty of the presented problem, an approach to solve it by using a Multi-Objective Evolutionary Algorithm (MOEA), the NSGA-II, was proven to be possible. The method found a good approximation to the set of Pareto solutions. The proposed MOEA approach successfully calcu-



(a) 50 scenarios



(b) 100 scenarios

Figure 4: Non-dominated solutions with high load.

lates good converter allocation and routing for a given set of requests known as scenarios which are used to model the uncertainty of future traffic.

In short, experimental results indicate that good solutions can be calculated and those solutions improve both, blocking probability and fairness, when the number of converter increases. At the same time, for a given number of converters, the blocking probability and unfairness are in trade-off, i.e. improving one deteriorates the other.

As future work, the authors are working to validate this proposal with larger networks and different topologies. At the same time, they are working on recommendations on how to choose a good solution from a Pareto set calculated with the proposed algorithm, considering always uncertain traffic.

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